Magnetoplasma Effects in Solids*

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Summary—Plasmas in solids show a more complex behavior than in gases since they reflect the symmetry properties of crystals. Since the carrier concentration has a wide range in semiconductors and metals, the plasma phenomena can be studied from microwaves to the ultraviolet. The effect of magnetic fields on the electromagnetic properties of plasmas has been experimentally investigated at microwave and infrared frequencies and has been utilized to measure dielectric constant and band structure of such solids in the limit of low magnetic fields. The magneto-plasma exhibits effects analogous to the galvanomagnetic phenomena. However, near resonance in the classical limit, they show up as depolarizing effects in semiconductors and also give rise to a new type of cyclotron resonance under anomalous skin conditions in metals.

Introduction

The electromagnetic properties of plasmas in ionized gases have been considered from the phenomenological classical magneto-ionic theory[1] in the linear limit for two sets of particles, *i.e.*, electrons and ions. Many of the phenomena, studied theoretically and observed experimentally in the various limits designated by Allis,[2] can also be found in solids. In addition, due to the nature of crystalline solids, these effects are further enriched by added complexities associated with the symmetry and band structure of the crystals. Still other differences arise from the dielectric properties and the greater electron densities which are possible. The latter may vary, for practical purposes, in a semiconductor from 10^6/cc to 10^{22}/cc for metals. Thus, in actuality, plasma effects in solids have been observed from microwave frequencies well into the vacuum ultraviolet.

Magnetoreflection

One of the first experiments performed was the ionospheric reflection from the E and F layers of the ionosphere.[3] If we neglect the collision of electrons and assume, therefore, that the medium is essentially dispersive, we can write the following expression for the complex index of refraction $(n+jk)$:

$$(n + jk)^2 = 1 - \frac{\omega_p^2}{\omega^2}; \qquad \omega_p^2 = \frac{Ne^2}{m\epsilon_0}, \tag{1}$$

where ω is the angular frequency of the electromagnetic wave; ω_p the plasma frequency; N, e, and m the density, charge, and mass, respectively, of the electron; and ϵ_0 the permittivity of free space. An electromagnetic wave incident normally on an idealized ionosphere will be reflected if its frequency is below a critical value, *i.e.*, $\omega < \omega_p$ since n is imaginary. Above this frequency, the medium can transmit the wave. The reflection coefficient $R = [(n-1)^2+k^2]/[(n+1)^2+k^2]$ plotted as a function of frequency has the form shown in Fig. 1 for both the ionosphere and a solid.

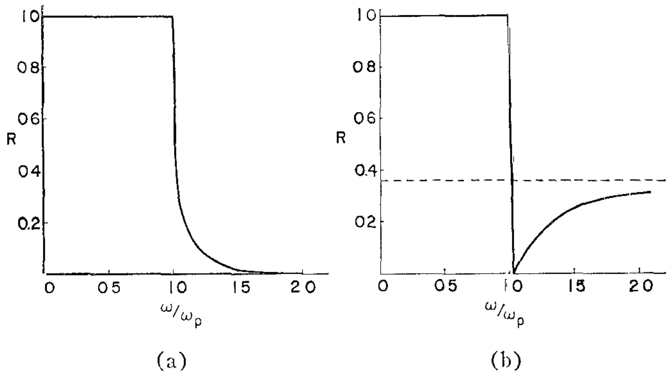

Fig. 1—Variation of the reflection coefficient R with frequency for (a) an idealized ionosophere, and (b) a solid.

Above the critical frequency ω_p the ionosphere is transparent and the reflection goes to zero. In case of a solid the situation is different, even for an isotropic plasma, since the material has a dielectric constant ϵ which differs from that of free space, so that

$$n^2 = \epsilon\left(1 - \frac{\omega_p^2}{\omega^2}\right), \qquad \omega_p^2 = \frac{Ne^2}{m^*\epsilon} \cdot \tag{2}$$

Thus, for frequencies just above the plasma frequency, the index of refraction can become unity, and the reflection is zero. Above this value the reflection increases to a higher value which at very high frequencies corresponds to the reflection of a pure dielectric with a dielectric constant ϵ. Another distinction between the solid and a gaseous plasma is that the mass of the carrier differs from that of the free electron and is designated by m^*, the effective mass. m^* is usually smaller than m, the free electron mass, although in a few cases it can also exceed this value. The position of the edge or the minimum $(n=1)$ has been utilized to determine m^* in semiconductors by measuring the carrier concentration N by the Hall effect and ϵ by reflection at short wavelengths in the infrared.

The situation becomes quite interesting when a magnetic field is applied. For the two cases of interest, the

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[1] E. V. Appleton, *Proc. Phys. Soc.* (*London*), vol. 37, pp. 16D; 1925.
D. R. Hartree, *Proc. Cambridge Phil Soc.*, vol. 25, pp. 47; 1929.

[2] W. P. Allis, "Propagation of waves in a plasma in a magnetic field," this issue pp. 79–82.

[3] E. V. Appleton and M. A. F. Barnett, *Nature*, vol. 115, pp. 333; 1925.
G. Breit and M. Tuve, *Phys. Rev.*, vol. 28, pp. 554; 1926.

index of refraction becomes

$$n_\pm^2 = \epsilon\left[1 - \frac{\omega_p^2}{\omega(\omega \mp \omega_c)}\right]; \qquad \omega_c = \frac{eH}{m^*c},$$

$$n_\perp^2 = \epsilon\left[1 - \frac{\omega_p^2(\omega^2 - \omega_p^2)}{\omega^2(\omega^2 - \omega_p^2 - \omega_c^2)}\right], \tag{3}$$

where ω_c is the cyclotron frequency, $n_\pm$ corresponds to the indexes of the right- and left-hand circularly polarized waves for propagation along the magnetic field, and $n_\perp$ is the index for a linearly polarized wave with the electric vector $\boldsymbol{E}$ perpendicular to the magnetic field $\boldsymbol{H}$ and propagating transverse to it. There is also an $n_\parallel$ for transverse propagation, which is given by (2) and corresponds to a linear polarization with $\boldsymbol{E}\|\boldsymbol{H}$. The plasma edge as shown in Fig. 1(b) is now split into two parts by the magnetic field. The condition is given by setting the indexes equal to zero. The results for the critical frequencies are identical for both the longitudinal and the transverse waves, since the propagation constant is zero. Hence, both physically and mathematically, there is no distinction. The critical or cutoff frequencies due to the plasma are then given by

$$\omega_\pm = \frac{\sqrt{\omega_c^2 + 4\omega_p^2} \pm \omega_c}{2}. \tag{4}$$

Eq. (4) intrinsically contains the essence of a variety of magnetoplasma phenomena which have been discussed in the literature. Perhaps the first, considered by Appleton and Hartree, is the splitting of the ionospheric reflection by the earth's magnetic field; such experiments were used in measuring its intensity. The conditions for the ionospheric case as well as that for many solids is that $\omega_c \ll \omega_p$. Hence, if (4) is expanded in terms of the magnetic field, we obtain

$$\omega_\pm = \omega_p \pm \frac{\omega_c}{2} + \frac{\omega_c^2}{8\omega_p}. \tag{5}$$

It is apparent that the plasma edge is split by the cyclotron frequency ω_c as depicted by the reflection spectrum shown in Fig. 2. The shape of the pattern for longitudinal and transverse propagation differs as shown. The latter is depicted for linear polarization, *i.e.*, $\boldsymbol{E}\perp\boldsymbol{H}$. In the case of solids, these effects can be used to measure the effective mass of electrons since the magnetic field and the frequency splitting of the edges can be determined. Indeed, such an experiment has been performed by Wright[4] in semiconductors with infrared radi-

[4] B. Lax and G. B. Wright, "Magnetoplasma reflection in solids," *Phys. Rev. Lett.*, vol. 4, pp. 16–18; January 1, 1960.

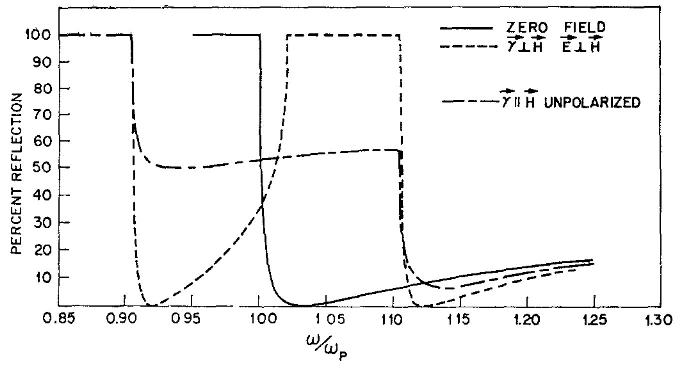

Fig. 2—Theoretical curves of magnetoplasma reflection for isotropic carrier and $\omega\tau \gg 1$. $\omega_c = 0.2\ \omega_p$. (Taken from Lax and Wright.[4])

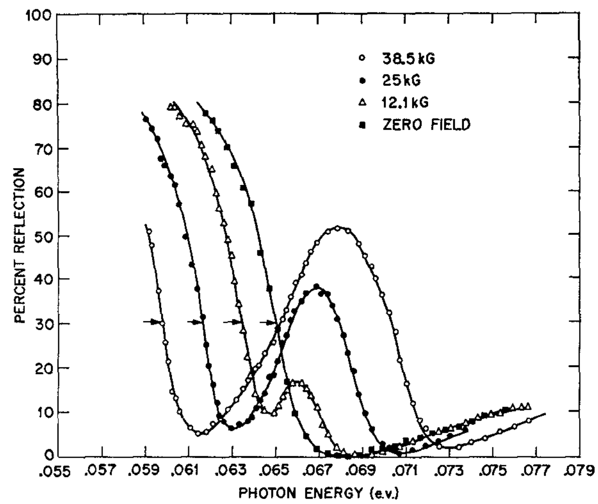

Fig. 3—Magnetoplasma reflection in *n*-type InSb. N—1.8×10^{18} cm^{-3}. (Taken from Lax and Wright.[4])

ation as shown in Fig. 3. From such experiments he has determined the effective masses of electrons in InSb and HgSe.

The phenomenon in this limit of magnetic field is of more basic interest for solids than is the determination of a simple effective mass. Actually, for somewhat higher fields, where the quadratic term of the magnetic field becomes important and the quadratic shift of the plasma edge can be observed, the magnetoplasma effect can be shown to be the dispersive analog of the classical galvanomagnetic phenomena. The latter can be discussed in terms of the component of current along an electric field, wherein the current density vector J is expanded in terms of the magnetic field as follows:

$$J_i = \sigma_i E_i + \sigma_{ij} E_i E_j + \sigma_{ijk} E_i H_j H_k + \cdots. \tag{6}$$

In this case, the first coefficient σ_i is the usual conductivity along the component of the electric field E_i. The second coefficient, which is linear in H, is associated with the Hall effect, and is used to determine the Hall mobility and the carrier concentration. The third coefficient which is quadratic in H is exhibited by the magnetoresistance of the solid. Thus, the galvanomagnetic effects can reflect the anisotropic properties of the crystal or that of the energy bands of the carriers which contribute to the conductivity. In an analogous manner, (6) can represent the current density of a plasma in a solid in the presence of an electromagnetic wave and a dc magnetic field. In this instance, however, the current vector, which is substituted into Maxwell's equations for evaluating the propagation constant, is assumed to be dispersive, *i.e.*, $\omega\tau \gg 1$, where τ, the scattering or collision time, is assumed large. Then the first term in (6) gives rise to the plasma frequency, the second term is associated with the linear splitting of the plasma edge and also with the Faraday rotation due to carriers in such materials, and the third term represents the quadratic shift of the plasma edges with magnetic field. Thus, the study of the magnetoplasma effects can reflect the anisotropy properties of the solids. One of the advantages of the magnetoplasma effects over the galvanomagnetic effects is that the latter measures conductivities or mobilities which are expressed as the ratio of the scattering time and the effective mass, *i.e.*, τ/m^*, whereas the magnetoplasma effects determine the effective masses directly. Thus, if there is any anisotropy in τ, this is not distinguished by the classical Hall effect or magnetoresistance.

To show how the magnetoplasma effects reflect the properties of the energy bands, we have worked out explicit results for electrons in germanium and silicon[4,5] whose motion is associated with spheroidal energy-momentum surfaces. The plasma frequency is given by

$$\omega_p^2 = \frac{Ne^2}{m^*\epsilon}; \quad m^* = \frac{3K}{2K+1} m_t; \quad K = m_l/m_t, \tag{7}$$

where m_t is the transverse effective mass of the elongated spheroid and m_l is the longitudinal mass, $m_l \gg m_t$. The frequency splitting of the plasma edge is isotropic in a cubic crystal such as germanium and silicon and is given by

$$\Delta\omega = \omega_t(K+2)/(2K+1), \tag{8}$$

where $\omega_t = eH/m_t c$ is the cyclotron frequency of the transverse mass. Thus, from the determination of the plasma edge and the splitting, one can determine m_t and K if the electron density N and the dielectric constant ϵ are known from independent measurements. The quadratic shift in a cubic crystal is not isotropic, and for a system of ellipsoidal surfaces in germanium and silicon for $\boldsymbol{E} \| \boldsymbol{H}$, it has the form

$$\Delta\omega = \frac{\omega_t^2}{\omega} \frac{(K-1)^2}{rK(2K+1)}, \tag{9}$$

where $r = 3$, 9, and 6 for H parallel to the [001], [111], and [110] crystal axes, respectively, in germanium; and $r = 3$ or 8 for H along the [111] or [110] axes in silicon. $\Delta\omega = 0$ for $H \|$ [001] in silicon. Analogous results can be obtained for the quadratic shift for $E \perp H$. The importance of these results is that if the quadratic effect is studied together with the linear effect, ω_t and K can be directly measured, and also the disposition of the family of ellipsoidal energy surfaces can be deduced from the anisotropy of the quadratic shift. The method can also be employed to study more complicated energy bands such as those associated with the holes in germanium and silicon.[6] The coefficients of (6) can be evaluated in terms of the band parameters in this case also. For noncubic crystals, information can be obtained from the plasma effects at zero magnetic field which can show anisotropy with crystal orientation and polarization of the electromagnetic waves. In bismuth such effects have been observed in the infrared on reflection[7] and are shown in Fig. 4. Additional information can be obtained if the magnetoplasma effect of these edges is studied in detail.

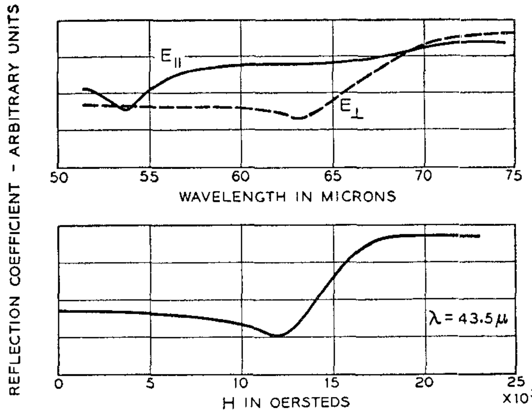

Fig. 4—(a) The reflection coefficient for a single crystal of pure bismuth with the surface normal to a twofold axis. $E_\perp$ and $E_\|$ refer to plane-polarized radiation perpendicular or parallel to a threefold axis. (b) The reflection coefficient, for the same sample as (a), shown as a function of magnetic field at a wavelength of 43.5 microns. The magnetic field is normal to the plane of the sample. (Taken from Boyle, Brailsford, and Galt.[7])

Cyclotron Resonance

Another limit of (6) which has been considered in connection with cyclotron resonance is the case where

[5] B. Lax and L. M. Roth, "Propagation and plasma oscillation in semiconductors with magnetic fields," *Phys. Rev.*, vol. 98, pp. 548–549; April 15, 1955.

[6] B. Lax and J. G. Mavroides, "Statistics and galvomagnetic effect in germanium and silicon with warped energy surfaces," *Phys. Rev.*, vol. 100, pp. 1650–1657; December 15, 1955.

J. G. Mavroides and B. Lax, "Magnetoresistance of holes in germanium and silicon with warped energy surfaces," *Phys. Rev.*, vol. 107, pp. 1530–1534; September 15, 1957.

[7] W. S. Boyle, A. D. Brailsford, and J. K. Galt, "Dielectric anomalies and cyclotron absorption in the infrared: Observations on bismuth," *Phys. Rev.*, vol. 109, pp. 1396–1989; February 15, 1958.

$\omega_c \gg \omega_p$. Usually, however, in such experiments, particularly at microwaves, the frequency is fixed and the magnetic field is varied. In this instance, only the positive index of refraction of (3) applies, since this is the one with the singularity corresponding to cyclotron resonance, *i.e.*, $\omega = \omega_c$. If, then, (3) is solved for the cyclotron frequency ω_c for $n_+ = 0$, we find

$$\omega_c = \omega - \frac{\omega_p^2}{\omega}\cdot \tag{10}$$

For frequencies near the plasma frequency but above it, this corresponds to cyclotron resonance; however, there is a plasma correction ω_p^2/ω which must be made to evaluate the effective mass. The above situation also holds for resonance in the infrared where the magnetic field is varied. Such experiments were carried out by Lipson[8] on InSb with fixed frequencies obtained by the use of reststrahlen plates which gave selected wavelengths in the far infrared centered at 63 μ, 83 μ and 94 μ. The plasma frequency, which at room temperature for the intrinsic carrier concentration of about $\sim 1 \times 10^{16}$/cc corresponds to a wavelength of $\sim$180 microns, produces a shift of the resonance frequency as the wavelength becomes larger, as shown in Fig. 5. Thus, at 94μ there is a correction of about 25 per cent to the resonance condition as shown by the intersection of the dotted line with the reflection curve.

In the infrared with a prism or a grating instrument, it is usual to vary the frequency; then, in the limit of $\omega_c \gg \omega_p$, (6) gives

$$\omega_1 = \omega_c + \frac{\omega_p^2}{\omega_c}, \qquad \omega_2 = \frac{\omega_p^2}{\omega_c}\cdot \tag{11}$$

The first of these equations is the equivalent of the previous result and gives the plasma correction for the cyclotron resonance condition. The second result represents a resonance which has been obtained in a somewhat different form by Dresselhaus, Kip and Kittel[9] which they designated as "magnetoplasma resonance." Their analysis involved resonance absorption in a sample of finite dimension, small compared to a wavelength or the penetration depth in the medium as limited by the plasma. This occurs when the plasma frequency is larger than that of the electromagnetic wave. The equation of motion of the carriers taking into account the depolarizing effect due to the plasma, which builds up charges on the surface of the specimen, can be written

$$m^*(\nu + j\omega)\boldsymbol{v} = q\boldsymbol{E}_i + q\boldsymbol{v} \times \boldsymbol{H}/c, \tag{12}$$

[8] H. Lipson, S. Zwerdling, and B. Lax, "Far infrared cyclotron resonance and magneto-plasma effects in I_nS_p," *Bull. Am. Phys. Soc.*, Ser. 2, vol. 3, p. 218; May 1, 1958.

[9] G. Dresselhaus, A. F. Kip, and C. Kittel, "Plasma resonance in crystals: Observations and theories," *Phys. Rev.*, vol. 100, pp. 618–625; October 15, 1955.

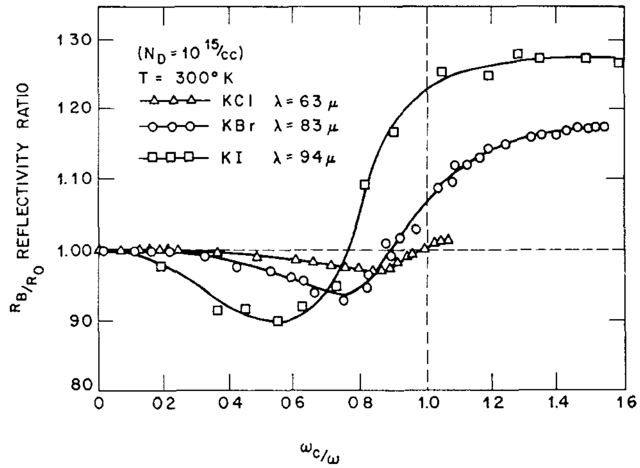

Fig. 5—Far infrared cyclotron resonance in InSb showing the magnetoplasma effect which shifts reflection minimum and crossover to lower fields. (Taken from Lipson, Zwerdling, and Lax.[8])

where ν is the collision frequency and q the charge of the carrier, and the internal field E_i is given in terms of the external field $\boldsymbol{E}$ by

$$(1 + \boldsymbol{L}\chi_0)\cdot \boldsymbol{E}_i = \boldsymbol{E} + j\boldsymbol{L}\cdot \boldsymbol{v}\,\frac{Nq}{\omega\epsilon_0}, \tag{13}$$

where χ_0 is the dielectric susceptibility, N the number of carriers, and $\boldsymbol{L}$ the depolarizing tensor. If (12) and (13) are solved for the velocity components, an effective conductivity tensor for a finite sample can then be obtained. From the tensor components it is possible to obtain expressions for the absorption of energy in specimens of different geometries. These results have been given for flat disks with magnetic field perpendicular, thin flat slabs with the field parallel, and long cyclinders with an axial field. For the first configuration, the magnetoplasma effect is eliminated. For the other two, the depolarizing factors produce interesting effects which can be most simply illustrated for the cylindrical geometry. If this is excited by circularly polarized waves, then the effective conductivities become

$$\sigma_\pm/\sigma_0 = \left[1 + j\left\{\omega - \frac{\omega_p'^2}{\omega} \mp \omega_c\right\}\gamma\right]^{-1}, \tag{14}$$

where

$$\sigma_0 = \frac{Ne^2\tau}{m^*} = \omega_{p0}^2\epsilon_0\tau$$

$$\omega_{p0}^2 = Ne^2/m^*\epsilon_0; \qquad \omega_p' = \omega_{p0}/\sqrt{2 + \chi_0}.$$

The absorption can then be calculated by taking the real part of the conductivities. The surprising result which evolves is that the negative circularly polarized wave interchanges roles with the positive one as the carrier density goes above a critical value $\omega_p' = \omega$. This is the condition for obtaining magnetoplasma resonance, and in the limit of high density where $\omega_p' > \omega$, the negative polarized wave has a resonance corresponding to

$$\omega\omega_c \approx \omega_p^2, \tag{15}$$

analogous to the low-frequency result of (11). The results are shown graphically for the real parts of σ_+ and σ_- for the cylinder in Fig. 6. For low densities $\omega_p' < \omega$, there is a resonance peak for the positive polarization at the usual cyclotron frequency. At the critical frequency $\omega_p' = \omega$, no resonance absorption occurs for either sense of polarization. However, at large densities $\omega_p' > \omega$, resonance occurs only for the negative circularly polarized wave. In this case, it can be shown that resonance occurs even when $\omega\tau < 1$. The condition for a resolvable peak requires that $(\omega_p^2/\omega)\tau = \omega_c\tau > 1$.

The magnetoplasma effects have been observed at microwave frequencies[9] in n-type InSb with the magnetic field both parallel and perpendicular to a finite disk, so that both $L_{\|}$ and $L_{\perp}$ had finite values. The "magneto-plasma resonance" absorption for H perpendicular to the sample is then given by

$$(\omega\omega_c)_{\perp} \approx \omega_{p0}^2 \frac{L_{\perp}}{1 + L_{\perp}\chi_0}. \tag{16}$$

When the magnetic field is parallel to the disk, the resonance condition becomes

$$(\omega\omega_c)_{\|} \approx \omega_{p0}^2 \sqrt{\frac{L_{\perp}L_{\|}}{(1 + L_{\perp}\chi_0)(1 + L_{\|}\chi_0)}}. \tag{17}$$

The ratio of the resonance fields for these two configurations is then obtained from the above and gives

$$\frac{\omega_{c\perp}}{\omega_{c\|}} = \sqrt{\frac{L_{\perp}(1 + 1L_{\|}\chi_0)}{L_{\|}(1 + L_{\perp}\chi_0)}}. \tag{18}$$

This ratio should be independent of the frequency, and for the results shown in Fig. 7 it has a value$\sim$0.8 with $L_{\perp} \approx 0.15$, $L_{\|} \approx 0.7$, and $\chi_0 = 15$; this is in reasonable agreement with the experimental results.

Anomalous Skin Effect

A phenomenon which can also be attributed to the existence of a high-density electron plasma is found in metals at low temperatures. The scattering time is rather long for pure metals, and therefore the medium at microwave frequencies can be considered to be dispersive; *i.e.*, $\omega\tau > 1$. Nevertheless, at these wavelengths the plasma behaves as a medium below cutoff or as a perfect reflector in which the penetration depth of the electromagnetic wave is given by $\delta \approx c/\omega_p$. For a metal such as copper, this penetration depth is of the order of 10^{-5} cm. In a metal such as bismuth, δ is of the order of 10^{-4} cm. The mean free path in these materials, however, is considerably longer at liquid helium temperatures, being of the order of 10^{-2} cm. Consequently, it is certain that electrons which participate in the interaction with the electromagnetic waves do so under a condition known as the anomalous skin effect. In the presence of a magnetic field, this results in a new and useful phenomenon which was first discovered by two Russian theoretical physicists, Azbel and Kaner.[10] According to their theory, if the magnetic field is parallel to the surface of the metal, some of the electrons move in a helical path cycle in and out of the skin depth con-

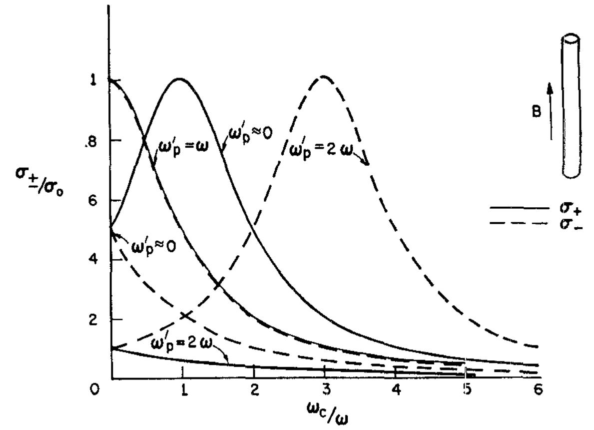

Fig. 6—Magnetoplasma effect for a circular cylinder with the magnetic field along the axis. Here $\omega\tau = 1$, $\omega' = \omega = \omega - (\omega_p')^2/\omega$ has the values shown. The absorption for linear polarization is proportional to $\sigma_+ + \sigma_-$. (Taken from an unpublished figure by B. Lax and L. M. Roth.)

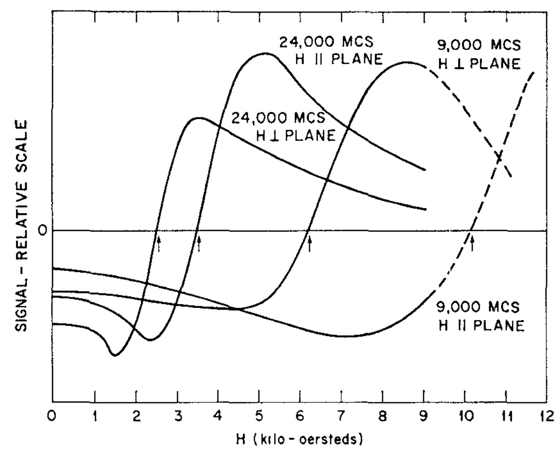

Fig. 7—Experimental plasma resonance absorption signals obtained with carrier modulation in a thin disk of n-type indium antimonide at 77°K, at 9000 Mc and 24,000 Mc as indicated. The static magnetic field is directed normal to, or parallel to, the plane of the disk in separate runs. The broken lines connect the curves below 9000 oersteds with single terminal points determined at higher fields. The resonance condition is determined approximately by the crossover points. (Taken from G. Dresselhaus, A. F. Kip, C. Kittel, and G. Wagoner, *Phys. Rev.*, vol. 98, pp. 556–557; 1955.)

[10] M. I. Azbel and E. A. Kaner, "The theory of cyclotron resonance in metals," *Soviet Phys. JETP*, vol. 3, pp. 772–774, December, 1956; "Theory of cyclotron resonance in metals," vol. 5, pp. 730–774; November, 1957.

taining the RF electric field, as shown in Fig. 8. If the magnetic field is tuned to resonance, then the electron and the electric field are in phase much similar to that in the cyclotron, where the skin depth now acts as do the dees in a cyclotron. This occurs when the magnetic field is tuned to the condition that $\omega_c=\omega$. However, the electrons can also show resonant absorption when the cyclotron rotation occurs in phase over several cycles of the RF field so that $n\omega_c=\omega$. Thus, this phenomenon gives rise to multiple or to subharmonic resonance absorption. The theory for the impedance of such absorption was worked out under the anomalous skin effect by using the Boltzmann transport theory.[10,11] The real part of this impedance which is proportional to the absorption is shown in Fig. 9. This phenomenon has now been observed on a number of metals at microwave frequencies[12] and the results, as observed in zinc, are shown in Fig. 10. Indeed, the behavior well confirms the theoretical predictions.

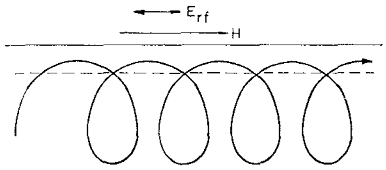

Fig. 8—Possible electron trajectory in a metal under the conditions of the Azbel-Kaner effect.

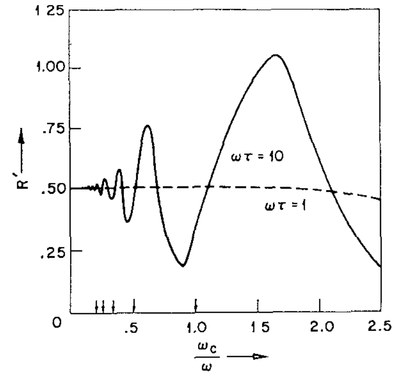

Fig. 9—Theoretical plot of $R'=R(16\pi\omega/3^{3/2}c^2)^{-1}(3\pi^2\sigma_0\omega/c^2v_F\tau)^{1/2}$, where R is the resistive component of the surface impedance, vs ω_c/ω for $\omega\tau=1$ and 10. The first five harmonics are indicated by arrows. The fundamental and first harmonic are appreciably shifted toward lower magnetic fields. This shift remains even for longer relaxation times. (Taken from Mattis and Dresselhaus.[11])

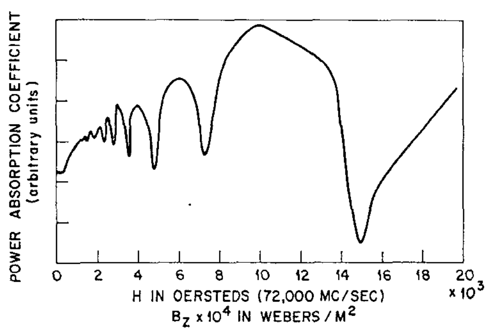

Fig. 10—Cyclotron resonance at 1.3°K in zinc with the magnetic field in the sample plane and along a sixfold axis. (Taken from Galt, Merritt, Yager, and Dail.[12])

Faraday Effect

The last phenomenon which we shall consider is well known to those who have worked in the field of microwave ferrites, *viz.*, the Faraday effect. Plasmas in a longitudinal magnetic field are capable of producing a measurable rotation in the spectral region from microwave frequencies into the infrared.[13] It has been found to be particularly useful for the study of semiconductors in the spectral range from a few microns to about 20 microns where polarizers and analyzers using optical techniques can be built. The theory is analogous to that of the ferrite case, and in the limit where the cyclotron frequency $\omega_c\gg\omega$ and the medium is dispersive, $\omega\tau\gg1$, which holds even at room temperature in the infrared; the result from classical theory is

$$\Theta = \frac{Ne^3H}{2\omega^2m^{*2}c^2\sqrt{\epsilon}}, \tag{19}$$

where Θ is the angle of rotation of a linearly polarized field propagating along a magnetic field, and the other parameters are those already defined in the text. The importance of this result is that it can be utilized in measuring m^* by determining N and ϵ by the Hall effect and reflection measurements, respectively, where other techniques are not feasible. Furthermore, the effective mass as a function of energy in the band can be

[11] D. C. Mattis and G. Dresselhaus, "Anomalous skin effect in a magnetic field," *Phys. Rev.*, vol. 111, pp. 403–411; July 15, 1958.
S. Rodriquez, "Theory of cyclotron resonance in metals," *Phys. Rev.*, vol. 112, pp. 1616–1620; December 1, 1958.
[12] E. Fawcett, "Cyclotron resonance in tin and copper," *Phys. Rev.*, vol. 103, pp. 1582–1583; September 1, 1956.
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A. F. Kip, D. N. Langenberg, B. Rosenblum, and G. Wagoner, "Cyclotron resonance in tin," *Phys. Rev.*, vol. 108, pp. 494–495; October 15, 1957.
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P. A. Bezuglyi and A. A. Galkin, "Cyclotron resonance in tin at 9300 Mcs," *Soviet Phys. JETP*, vol. 6, pp. 831–833; April, 1958.
J. K. Galt, F. R. Merritt, W. A. Yager, and H. W. Dail Jr., "Cyclotron resonance effects in zinc," *Phys. Rev. Lett.*, vol. 2, pp. 292–294; April 1, 1959.
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studied by varying the concentration N. Such studies have been carried out by Smith, *et al.*[14]

Conclusion

A variety of phenomena involving the behavior of a plasma in a solid in the presence of a magnetic field has been described. With almost no exception, each of these in one form or another reflects the band properties of the semiconductor or metal which is being investigated. Although the phenomena are complex and of primary interest to physicists who wish to measure the fundamental parameters associated with holes and electrons in these materials, the results are, nevertheless, already of some practical significance to engineers who wish to utilize these effects for developing new kinds of devices. Obviously, these magnetoplasma effects can be utilized as polarizers and for nonreciprocal components in the regions of the far infrared spectrum where such devices do not exist. However, this type of investigation is attractive because it may be instrumental in the development of an infrared cyclotron resonance maser. The magnetoplasma effects permit not only the investigation of the anisotropy of the effective masses, but also their variations with energy, an important requirement for the development of such a cyclotron resonance maser.[15]

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Coherent Excitation of Plasma Oscillations in Solids*

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Summary—**Considerations are put forth concerning the feasibility of observing the coherent excitation of plasma oscillations in a two-component plasma of electrons and holes in semiconductors or semimetals. By coherent excitation is meant the onset of a high-frequency ("two-stream") instability arising from an appreciable drift of electrons vs holes under the action of an applied electric field. Conditions favorable to coherent excitation include a sizeable difference in electron and hole masses, and long relaxation times for both kinds of particles. The extent to which such conditions are present in InSb is discussed.**

THE PLASMA formed by the electrons and holes in a semiconductor or semimetal offers, in many respects, a promising "laboratory" for carrying out experiments of interest on collective properties of plasmas. By gaseous standards, the plasma is well behaved. One can measure and vary in simple fashion the concentrations, mass ratios, and temperature ratios of the two plasmas. The principal drawback to carrying out plasma experiments is that the electrons and holes in this fully ionized plasma are not completely free; they are scattered by phonons, impurity atoms, and, in some cases, one another, to an extent which may be important for the study of collective phenomena. Indeed, if ω is the frequency of interest for the phenomenon under investigation, and $\tau_{\pm}$ represents the electron (or hole) relaxation time associated with the scattering mechanisms, then it is necessary that

$$\omega \tau_{\pm} \gtrsim 1,$$

in order that the collective behavior be observable.

In the present paper some theoretical investigations of collective behavior in solid-state plasmas, which have been carried out in collaboration with J. R. Schrieffer,[1] will be summarized. The problem of particular concern was the feasibility of observing in such plasmas a high frequency instability associated with the coherent excitation of plasma oscillations. The instability, which resembles the "two-stream" instability encountered in electron beam studies, arises if a sufficiently large drift of electrons vs holes is produced under the action of an applied electric field.

Most previous studies[2] of instabilities in the two component plasmas were carried out under the assumption that the electron and ion (or hole) temperatures were equal. In these circumstances the required drift velocity is of the order of $1.3v_{-}$, where v_{-} is the electron thermal

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